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# Preferences, Consumption Smoothing, and Risk Premia \*

by

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### Abstract

Risk premia in the consumption capital asset pricing model depend on preferences and dividend. We develop a decomposition which allows a separate treatment of both components. We show that preferences alone determine the risk-return tradeoff measured by the Sharpe-ratio. In general, the risk-return trade-off implied by preferences depends on the elasticity of a preference-based stochastic discount factor for pricing assets with respect to the consumption innovation. Depending on the particular specification of preferences, the absolute value of this elasticity can coincide to the inverse of the elasticity of intertemporal substitution (e.g. for habit formation preferences) or the coefficient of relative risk-aversion (e.g. for Epstein-Zin preferences). We demonstrate that preferences based on a small elasticity of intertemporal substitution, such as habit formation, produce small risk premia once agents are allowed to save. Departing from the complete markets framework, we show that uninsurable risk can only increase the Sharpe-ratio and risk premia if dividends are correlated with individual consumption.

# 1 Introduction

Risk premia derived from the consumption capital asset pricing model (CCAPM) are determined by two parts: preferences and dividends. In most of the literature these two concepts have been entangled because dividends have been modeled as claims to consumption. Besides the lack of a better model for dividends in an aggregate economy, there is no justification to equate dividends with aggregate consumption. On the contrary, this modeling choice has led to a confusing interpretation of the role of consumption in determining risk premia. In this paper we provide a decomposition of the role of preferences and dividends. We show that preferences alone determine the position of the capital market line in the conditional mean - standard deviation diagram. Dividends are responsible for the position of a given asset on or below the capital market line. We argue that the Sharpe-ratio defined as the slope of the capital market line summarizes the risk - return tradeoff implied by preferences. We derive a simple way to compute the Sharpe-ratio for any preference specification: the Sharpe ratio depends on the product of the standard deviation of the consumption innovation with the elasticity of a preference-based stochastic discount factor (PSDF) for pricing assets with respect to the innovation in consumption. Hence, for any general specification of preferences the only parameter affecting risk premia is this preference factor elasticity. To match the observed Sharpe-ratio in post-war US data of about 0.27, the elasticity has to equal about -50.

We demonstrate how this PSDF elasticity is related to more conventional preference parameters for different utility functions. For standard time-separable power utility, that PSDF elasticity equals the negative of the coefficient of relative risk aversion and the inverse of the elasticity of intertemporal substitution. For power preferences with habit formation, that PSDF elasticity equals the negative inverse of the elasticity of intertemporal substitution but is not directly related to relative risk aversion. For recursive Epstein-Zin preferences, that PSDF elasticity is related to relative risk aversion but not to the elasticity of intertemporal substitution. This shows that neither risk aversion nor intertemporal substitution in general determine risk premia.

No matter what preferences are assumed, a very negative PSDF elasticity of about -50 is required to match the observed Sharpe-ratio. This has important implications. We will argue below that that a satisfactory solution for the equity premium puzzle (or the Sharpe-ratio puzzle, as we define it) which depends on preferences of an representative consumer should fulfill the following additional criteria. For a given set of parameters, these preferences should imply a Sharpe-ratio of about 0.27 which requires a PSDF elasticity of about -50, they should not imply an extremely low elasticity of intertemporal substitution and they should not rely on extremely high risk aversion. The elasticity of intertemporal substitution controls the desire to smooth consumption between periods. If

the elasticity of intertemporal substitution is very low, consumers will be very reluctant to adjust consumption over time and hence will imply that consumption will be too smooth as soon as consumers are allowed to save (which they are not in the Lucas (1978) exchange economy). This will in turn imply a low Sharpe ratio. We regard a coefficient of relative risk aversion of around 50 as equally unpalausible. While it is hard to estimate risk aversion directly, Barsky, Kimball, Juster and Shapiro (1995) present survey evidence. They find that most individuals are very risk averse with an average risk aversion across individuals of about 4. Moreover, extremely risk averse agents would be willing to pay large amounts to avoid atemporal gambles. A simple calculation shows that an agent with relative risk aversion of 50 would be willing to pay 8.7% of her wealth to avoid a 50-50 gamble to win or loose 10% of her wealth.

None of the preferences considered passes all three criteria. Habit formation, while allowing for low risk aversion, relies on a low elasticity of intertemporal substitution. Epstein-Zin preferences require a high risk aversion and time-separable power preferences fail on both grounds. Perhaps, a combination of habit formation and Epstein-Zin preferences has the potential to be consistent with our criteria.

We furthermore show that standard dividend processes are not volatile enough to produce sizeable risk premia, even when we assume that preferences imply a Sharpe-ratio of one quarter. Following the tradition of modeling aggregate dividends as claims to aggregate consumption, we show that (log) consumption is not volatile enough by a factor of about 13 to account for the observed equity premium, even given a Sharpe-ratio of 0.27. In other words, puzzles about risk premia can be decomposed into a puzzle about the Sharpe-ratio and a puzzle about dividend processes. The Sharpe-ratio puzzle only concerns preferences and is completely independent of the dividend puzzle. Hence different research agendas are required to address the two separate puzzles.

In extensions to the basic model, we demonstrate that our setup can easily be extended to allow for multiple factors (e.g. to model inflation risk for bonds). There is empirical evidence that the Sharpe-ratio is time-varying, for example due to business cycle movements. We show that this can be modeled either by assuming a heteroskedastic consumption or by generating a time-varying preference factor elasticity. Campbell and Cochrane (1996) present such a model.

Another application of the Sharpe-ratio criterion are models with idiosyncratic risk and uninsurable income shocks. These models have received a lot of attention lately as a possible way to generate large risk premia (Weil (1992), Lucas (1994), Heaton and Lucas (1996), Constantinides and Duffy (1996), Den Haan (1996)). We show that the Sharpe-ratio only increases when there are assets with dividends which are correlated with the *idiosyncratic* component of consumption. Since this component is assumed to be uninsurable, it is questionable whether these assets exist. Hence, it is not obvious how these models can generate a larger Sharpe-ratio and risk premia.

The analysis could be sharpened in a number of ways. First, we take the point estimates as given without taking account estimation standard errors. Second, the estimates represent unconditional moments while the theory is written in terms of conditional moments. Intuition suggests that it is even harder to fit the model using conditional estimates, but this is also worth investigating more closely. On the theoretical side, we deal only with risk premia. Preferences also determine the risk-free rate. Weil (1989) argues that there is also a risk-free rate puzzle which we do not address.

The remainder of the paper is organized as follows. Section 2 presents some stylized facts concerning risk premia and the Sharpe-ratio. Section 3 shows the decomposition of risk premia into the Sharpe-ratio and dividends. Section 4 studies some preference specifications which have been proposed in the literature using our decomposition. Section 5 shows that preferences which generate high Sharpe-ratios imply that consumption becomes extremely smooth once there are some smoothing possibilities. Section 6 looks at how models with idiosyncratic shocks fare in terms of the Sharpe-ratio and section 7 concludes.

## 2 The Facts

Table 1 summarizes some important and well-known facts about asset markets. The data are sampled at quarterly frequency and range from 1/48 to 1/96. Returns are real and reported in per cent. T-bills have a low mean return and do not vary much over time. Long Government bonds have a slightly higher average return than T-bills but are much more volatile. Stocks, measured here as the SP500 index, have a substantially higher mean return than bonds and are also more volatile. Asset premia are returns minus returns to T-bills. The equity premium is much higher than the long bond premium. The ratio of the average return premium divided by the standard deviation of the return premium is much higher for stocks than for long bonds. In other words, stocks deliver a higher return per unit of standard deviation.

Figure 1 shows the return data in the standard textbook mean-standard deviation frontier. It is standard practice to use a broad stock market index as an approximation for the market portfolio. Hence the capital market line (CML) is the ray starting at the T-bill point through the SP500 point. The slope is known as the Sharpe-ratio (or the price of risk). In postwar quarterly data, the Sharpe-ratio is 0.27. Note, that this is a lower bound of the Sharpe-ratio, since it is defined as the maximal ratio of mean return to standard deviation. There might very well exist an asset with a higher price of risk than the SP500, for example an index of small stocks. For the remainder of this paper, we take a Sharpe-ratio of 0.27 as a fact.

Mehra and Prescott (1985) have shown that standard models are not able to produce a large enough wedge between risky and riskless assets. They were concerned with

Table 1: Asset Market Facts  
 Real Returns, US data, % per quarter, 1/1948-1/96

Asset	Mean	Std. Deviation	Mean/Std. Deviation
T-bills	0.19	0.86	
Long Gov't Bonds	0.37	4.83	
SP500	2.17	7.53	
Long Gov't Bond Premium	0.18	4.75	0.038
Equity Premium	1.99	7.42	0.268

Note: Returns are measured at quarterly frequency. Units are per cent per quarter. Risk premia are computed as the difference between the asset return and the T-bill rate. Source: Ibbotson Associates

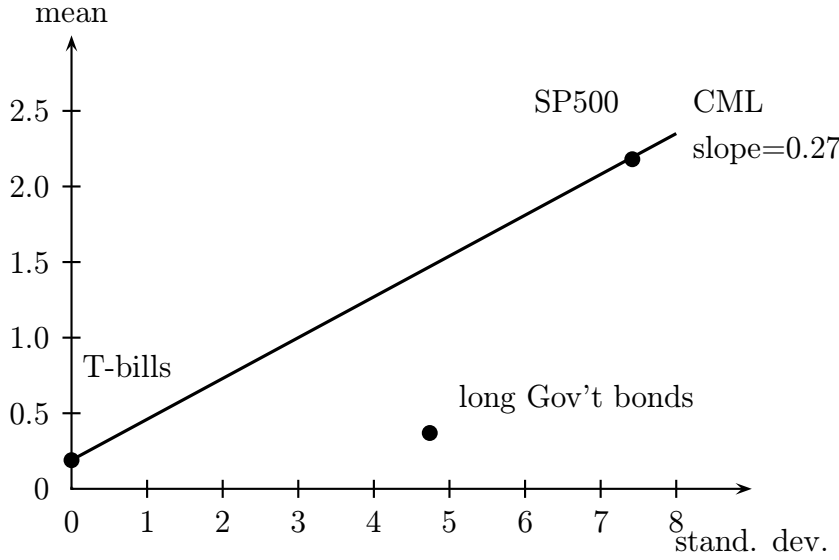
matching the first moment of the equity premium. Much of the literature following Mehra and Prescott (1985) has continued in that fashion. In this paper, we argue that the ratio of the first and second moments is even more important than just the first moment. In other words, we focus of the risk-return tradeoff implied by a model. In terms of the mean-standard deviation diagram, we decompose the equity premium into two parts. First, we study which features of the model determine the slope of the capital market line. Given the position of the CML, we ask what determines the position of assets on (or below) the CML. This decomposition will turn out to be extremely useful to evaluate models regarding their risk-return implications.

Table 1 and Figure 1 present the *unconditional* moments of asset returns. In other words, these moments are simple sample averages. In contrast, the theory laid out below will be written in terms of *conditional* moments. It is important to keep this difference in mind. We do not attempt a serious investigation of this issue, however, one can see intuitively how Figure 1 will be affected once conditional moments are considered. It is well-known that it is difficult to forecast mean returns of assets while squared returns are forecastable, for example using GARCH processes. This implies that the CML for conditonal asset returns will be *steeper* than for unconditional returns. Hence, the unconditional Sharpe-ratio can be viewed as a lower bound compared to a more serious investigation of conditional moments. For now, we take a Sharpe-ratio of 0.27 as given keeping in mind that the the conditional Sharpe-ratio is probably higher.

### 3 Decomposing Risk Premia

Risk premia based on on the consumption CAPM modeled have two separate components: preferences and dividends. In this section we provide a framework which allows

Figure 1: Mean-Standard Deviation Frontier: Quarterly Data



Note: CML with postwar quarterly data from Table 1.

a sharp decomposition of these parts. The approach is in spirit of the Hansen and Jagannathan (1991) volatility bounds. However, we make a slightly more restrictive distributional assumptions to obtain an analytical framework which lends itself better to economic interpretation. The key assumption in addition to Hansen and Jagannathan is that the innovation in consumption are lognormal, but not necessarily i.i.d.. We show that preferences determine the position of the capital market line in the (conditional) mean-standard deviation diagram. Given the capital market line, dividends determine the position of a given asset on (or below) the capital market line.

### 3.1 Consumption-based Asset Pricing

To talk meaningfully about consumption-based asset pricing in general, we envision the following framework. Let  $(S_t)_t$  be a Markov process on some state space and let the consumption<sup>1</sup>  $C_t \in \mathbf{R}$  of some agent under consideration<sup>2</sup> be determined by a function  $C_t = C(S_t)$  of the state  $S_t$ . Let  $Z_t \in \mathbf{R}^{n_z}$  denote “summary” information about past consumption, evolving according to a summarizer function  $g$ :

$$Z_t = g(C_t, Z_{t-1})$$

<sup>1</sup>Note, that we concentrate on the case of a single consumption good. The case of multiple consumption goods is not hard to treat as well, see appendix A for details.

<sup>2</sup>We do not yet presume that this agent is representative.



Let  $V(S_t, Z_{t-1}) \in \mathbf{R}$  be the “value function” of the remaining stream of consumption as of date  $t$ , evolving according to

$$V(S, Z) = f(C(S), Z, E[h(V(S', g(C(S), Z))) \mid S], S) \quad (1)$$

where  $f(C, Z, E, S) \in \mathbf{R}$  is an aggregator function and  $h(V) \in \mathbf{R}$  is a (typically monotone) transformation of  $V$ . The agent prefers higher values of  $V(\cdot, \cdot)$  to lower values. This framework is discussed in greater detail in appendix A. There, it is shown how all the particular utility functions discussed in this paper can be written in this form. Furthermore, assuming existence and differentiability, the asset pricing formula

$$E_t[M_{t+1}R_{t+1}] = 1, \quad (2)$$

is derived (see equation (45)). This is the well-known Lucas (1978) asset pricing formula, generalized to the context here. As for notation:  $E_t[\cdot]$  is short-hand for the conditional expectation  $E[\cdot \mid S_t]$ .  $M_{t+1}$  is a function of past, present and future consumption of the agent as well as the states  $S_t$  and  $S_{t+1}$ , which is measurable with respect to the information of date  $t + 1$ . It is rigorously derived in A.2 and defined in equation (44).  $R_{t+1}$  is the gross return on any asset from period  $t$  to  $t + 1$ . For example, if some asset is traded at price  $P_t$  in period  $t$  after paying dividends  $D_t$ , one has

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}. \quad (3)$$

The factor  $M_{t+1}$  is a *preference-based stochastic discount factor* (PSDF), which one might think of as a generalized version of the intertemporal marginal rate of substitution. In fact, it coincides with the intertemporal marginal rate of substitution for the standard case of a time separable utility function (i.e. for the case in which the aggregator function  $f(C, Z, S, E)$  is linear in  $E$  and independent of  $Z$ ). However, we would like to avoid calling it the intertemporal marginal rate of substitution also in this general setting. Such a terminology might be misleading because the “substitution experiment” here is one of one unit of consumption at date  $t$  versus a bundle of consumption at date  $t + 1$ , namely one unit for each contingency. Thus, attitudes towards risk aversion as well as attitudes towards intertemporal substitution matter for the agent in evaluating this “experiment” and thus, they matter for  $M_{t+1}$ . Separating these two from each other is precisely the point of the work by e.g. Epstein and Zin (1989, 1991), and calling  $M_{t+1}$  the “intertemporal marginal rate of substitution” could be misunderstood as a claim that it is only intertemporal substitution which matters. Furthermore, for some preferences, certain coefficients have been labeled to represent the elasticity of intertemporal substitution: it may thus be confusing if we called  $M_{t+1}$  the marginal rate of intertemporal substitution and then tried to explain, why it is occasionally unrelated to that elasticity. To avoid this potential confusion, we call  $M_{t+1}$  the preference-based stochastic discount factor for pricing asset.

What matters, for our analysis however, is the fact that  $M_{t+1}$  is *independent* of the asset under consideration. This important property is proved in A.2. Thus, equation (2) shows that the stochastics underlying asset pricing can be broken into two, unrelated pieces: the stochastics coming from the asset return and embodied in  $R_{t+1}$  and the stochastics from the consumption process and its utility evaluation by the agent, embodied in  $M_{t+1}$ . This insight is the key building block of our analysis.<sup>3</sup>

To proceed with our analysis, it is useful to utilize loglinear approximations. Let lower cases denote the logarithm of a variable, e.g.  $c_t = \log C_t$ . Without loss of generality we can decompose log consumption into its conditional expectations and its innovation

$$c_{t+1} = E_t c_{t+1} + \epsilon_{t+1}. \quad (4)$$

The substantive assumption we make is that the innovations in log consumption are normal:  $\epsilon_t \sim N(0, \sigma_{\epsilon,t}^2)$ . Note that the variance is allowed to change over time.

A key issue is the interpretation of (4). Lucas (1978) assumes a pure exchange economy to derive (2). Hence, consumers cannot smooth consumption over time and are forced to consume their exogenously given endowment. In other words consumption can be modeled as being exogenous to the model and  $\epsilon_t$  is the ‘deep’ shock. However, more realistic models allow consumers to choose consumption based on some more fundamental shock to the economy. These smoothing channels might include storage, savings, and labor input. We will contrast models with these features with the Lucas (1978) model below. Nevertheless, the asset pricing calculations *given* a consumption process are valid in any model.

For the rest of the paper we assume that the preference-based stochastic discount factor for pricing assets depends only on consumption. This is in line with most of the CCAPM literature but it is restrictive. Many macroeconomic models assume the utility also depends on the amount of leisure enjoyed in a period. Abstracting from leisure we can decompose the log PSDF in a similar fashion as log consumption as

$$m_{t+1} = E_t m_{t+1} + \eta_{m\epsilon,t+1} \epsilon_{t+1}. \quad (5)$$

$\eta_{xy}$  denotes the elasticity of variable  $x$  with respect to variable  $y$ .<sup>4</sup>  $\eta_{m\epsilon}$  represents the elasticity of the PSDF with respect to the consumption innovation  $\epsilon$  and plays a crucial role for asset prices. This elasticity is determined by the preferences. For time-separable CRRA preferences, we have  $m_{t+1} = -\gamma \Delta c_{t+1}$  where  $\gamma$  is the coefficient of relative risk aversion (RRA). Hence  $\eta_{m\epsilon} = -\gamma$ .<sup>5</sup> However, for more general preferences, such as

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<sup>3</sup>Of course, it also is the key building block in the Hansen-Jagannathan analysis of consumption-based capital asset pricing models, see Cochrane and Hansen (1982).

<sup>4</sup>In general the elasticities and innovation-variances can be time-varying, i.e. they have a time subscript. In order to avoid excessive notation, we will usually leave away the time subscript. However, the reader should keep in mind that our setup allows for this.

<sup>5</sup>It is easily shown that for any *time-separable* preferences,  $\eta_{m\epsilon} = -\text{RRA} = -1/\text{EIS}$ .

habit formation,  $\eta_{m\epsilon}$  will not equal the RRA as we will show below. One of the key insights is that  $\eta_{m\epsilon}$  is the *only* preference parameter relevant for risk premia. Note, that we allow for a time-changing elasticity as well which might arise due to time-inseparable preferences (see Campbell and Cochrane (1996) for such a model).

To compute the price of an asset we need to specify its dividends. In principle any dividend stream can be priced using equation (2). A real bond is defined as having  $D_t = 1$ . A claim to consumption (which often is used as ‘equity’ in exchange economies) has  $D_t = C_t$ . In models with an explicit production sector dividends to equity are usually the rental rate of capital, which depends on the marginal product of capital. The second component of any asset is its maturity. A short real bond pays a dividend only in the next period, long bonds pay for several periods. For now, we only consider assets with dividends which only depend on the consumption stream, this will be relaxed later (to allow for example for inflation risk). Decompose the log return of an asset in its conditional expectation and innovation:

$$r_{t+1} = E_t r_{t+1} + \eta_{r\epsilon} \epsilon_{t+1}. \quad (6)$$

Since asset dividends are assumed to be affected only by innovations in consumption, the unexpected return depends only on  $\epsilon_{t+1}$ .  $\eta_{r\epsilon}$  is the elasticity of the return with respect to the consumption innovation and depends on the dividend stream of the asset. For example, the return of a riskless real 1-period bond is unaffected by  $\epsilon$  and therefore has  $\eta_{r\epsilon} = 0$ . Hence the log risk-free rate equals minus the expected log PSDF:  $r_{t+1}^f = -E_t m_{t+1}$ .  $\eta_{r\epsilon}$  equals unity for a 1-period claim to consumption. Assets with longer maturity can have more complicated  $\eta_{r\epsilon}$  because of capital gains.

Using the normality assumption of  $\epsilon$  we can use equation (2) to express the expected return of an asset as <sup>6</sup>

$$E_t r_{t+1} = -E_t m_{t+1} - (\eta_{r\epsilon} + \eta_{m\epsilon})^2 \sigma_\epsilon^2 / 2. \quad (7)$$

Let  $r_{t+1}^e = \log E_t R_{t+1}$  be the logarithm of the expected gross return. <sup>7</sup> Equation (7) can be used to calculate the conditional expected risk premia  $r_{t+1}^{rp}$

$$r_{t+1}^{rp} \equiv r_{t+1}^e - r_{t+1}^f = -\sigma_{mr,t} \quad (8)$$

$$= -\eta_{m\epsilon} \eta_{r\epsilon} \sigma_\epsilon^2, \quad (9)$$

where  $\sigma_{mr,t}$  is the conditional covariance between the log PSDF and the log asset return. Equation (8) is the well known consumption CAPM (CCAPM) written in logs. Assets with a high conditional covariance with the PSDF carry a low risk premium. In this model the conditional covariance is the negative of a product of three parts. First, the

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<sup>6</sup>For normal  $x$ ,  $E(e^x) = e^{E(x) + .5 \text{Var}(x)}$ .

<sup>7</sup>Note, that in general  $r_{t+1}^e \neq E_t r_{t+1}$  because of Jensen’s inequality. Using this notation, we avoid cluttering the equations with variance terms which are due to Jensen’s inequality.

elasticity of the PSDF with respect to the consumption innovation. This number depends only on the preferences of the consumer and will be the same for different assets. Second, the elasticity of the asset return with respect to  $\epsilon$  which depends on the dividend stream of the asset.

As demonstrated by Mehra and Prescott (1985), the risk premia generated by this model are much smaller than risk premia in real financial markets. Since risk premia are composed of preferences and dividends, it is useful to decompose the effect of the two parts. This insight was the motivation of Hansen and Jagannathan (1991) to study volatility bounds of the PSDF. The drawback of their approach is that it is difficult to obtain economic intuition regarding their bounds. Given our lognormal model we obtain simple analytical expressions for the role of preferences and the role of dividends.

### 3.2 The Role of Preferences

In this subsection we show that preferences alone determine the position of the capital market line (CML) in the conditional mean-standard deviation frontier. The CML restricts the position of asset in the mean-standard deviation frontier since all assets by definition have to lie below it. Hence the slope of the CML, usually referred to as Sharpe-ratio or price of risk, is a useful measure of the risk-return tradeoff of preferences. Note, that dividends of specific assets play no role here. Therefore the role of preferences concerning risk premia can be summarized by the CML.

To compute the CML we require the risk-free rate and the Sharpe-ratio. The risk-free rate is simply the inverse of the expected PSDF:  $R_{t+1}^f = 1/E_t M_{t+1}$ . The Sharpe-ratio is given as (following Campbell and Cochrane (1996) and using (2)):

$$SR_t = \max_{\text{all assets}} \frac{E_t [R_{t+1} - R_{t+1}^f]}{\sigma_t [R_{t+1}]} \quad (10)$$

$$= \max_{\text{all assets}} -\rho_t(M_{t+1}, R_{t+1}) \frac{\sigma_t [M_{t+1}]}{E_t [M_{t+1}]} \quad (11)$$

$$= \frac{\sigma_t [M_{t+1}]}{E_t [M_{t+1}]}, \quad (12)$$

where  $\sigma_t$  denotes the conditional standard deviation operator and  $\rho_t$  the conditional correlation operator. Note that one asset with  $\rho_t = -1$  is a 1-period claim to consumption.

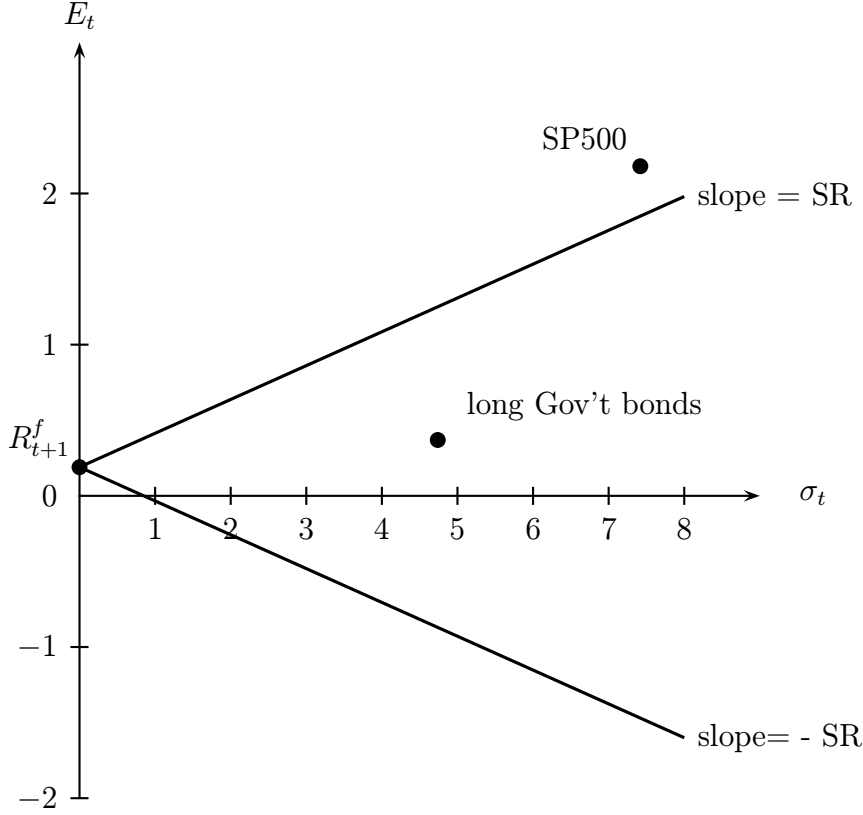
The Sharpe-ratio is solely a function of the first and second conditional moments of the PSDF. It is important to realize that it is independent of any particular asset. The properties of the PSDF are sufficient to calculate the risk-return tradeoff of the preferences. In our model we can write the Sharpe-ratio as follows (again using the normality assumption of  $\epsilon$ ):<sup>8</sup>

$$SR_t = \left( e^{\sigma_{m,t}^2} - 1 \right)^{1/2} \quad (13)$$

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<sup>8</sup>If  $x$  is normal, then  $\sigma(e^x)/E(e^x) = (e^{\sigma_x^2} - 1)^{1/2}$ .

Figure 2: Mean-Standard Deviation Frontier Implied by Preferences



Note: Figure shows the conditional mean-standard deviation frontier implied by time-separable CRRA preferences with  $\gamma = 40$ .

$$\approx -\eta_{m\epsilon}\sigma_{\epsilon}, \quad (14)$$

where  $\sigma_{m,t}^2$  denotes the conditional variance of the log PSDF.<sup>9</sup> All standard preferences will imply a negative elasticity of the PSDF with respect to a positive consumption innovation, hence the solution in equation (14) represents the upper ray in the mean-standard deviation diagram. The Sharpe-ratio is the product of the negative elasticity of the PSDF with respect to  $\epsilon$  and the standard deviation of  $\epsilon$ . The Sharpe-ratio together with the risk free rate tells us the maximal risk premium of an asset for a given standard deviation. All assets, independent of their dividend stream, are positioned in the cone formed by the two rays starting at the risk-free rate with slope  $SR_t$  and  $-SR_t$  in Figure 2. Figure 2 shows the CML for CRRA preferences with  $\gamma = 40$  and  $\sigma_{\epsilon} = 0.56\%$ .

<sup>9</sup>The Sharpe-ratio in the data is 0.27 (see Table 1). For small number like this, the approximation is very precise:  $(e^{(.265^2)} - 1) \cdot .5 = .27$ . Alternatively, we could define risk premia as  $E_t[R_{t+1}]/R_{t+1}^f$ . Then (14) is the exact expression for the Sharpe-ratio.

A simple calculation shows that standard time-separable CRRA preferences require a very high risk aversion (or, equivalently, an elasticity of intertemporal substitution close to zero) to create a large enough risk-return tradeoff. Recall from Table 1 that the ratio of the mean to the standard deviation of the equity premium is 0.27 in quarterly data. Hence the Sharpe-ratio in the data is at least 0.27. For time-separable CRRA preferences  $\eta_{m\epsilon} = -\gamma$ , where  $\gamma$  is the coefficient of relative risk aversion. The standard deviation of the innovation in log consumption is 0.56% in quarterly postwar US data (see Campbell and Cochrane (1996)). Hence a risk aversion of 48 is required to match the point estimate for the Sharpe-ratio. Figure 2 shows the mean-standard deviation diagram for  $\gamma = 40$ . The SP500 is located outside the admissible region. Note that the long bonds are well inside the cone.

This demonstrates that standard preferences require a very high risk aversion, as already noted by Mehra and Prescott (1985). Risk aversion coefficients of around 50 are usually regarded as implausible. As a response to this result, many alternative preference specifications have been considered in the literature. We will evaluate these using our Sharpe-ratio criterion below. However, note that the *only* preference parameter which affects the risk-return tradeoff of any preference specification is the elasticity of the PSDF to the consumption innovation.

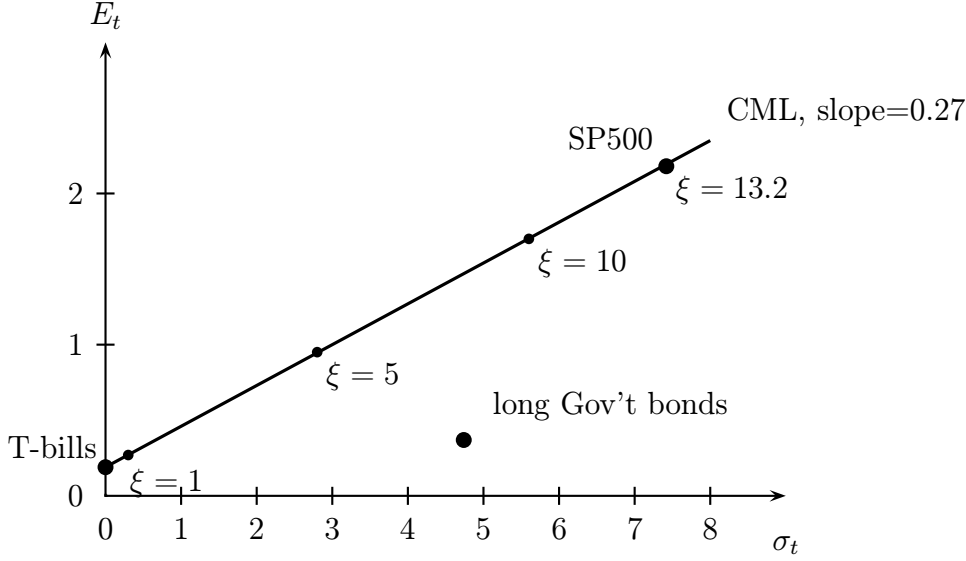
### 3.3 Dividends

After studying the role of preferences in determining risk premia, we now look at dividends. In most of the CCAPM literature aggregate dividends are modeled as claims to aggregate consumption. From a theoretical perspective this is clearly an unsatisfactory approach. However, the underlying Lucas exchange economy does not allow for a more refined definition of dividends. Even in models with an explicit production sector it is not obvious how to model dividends. One reason is that the optimal capital structure of profit maximizing firms is undetermined. So leverage becomes essentially a free parameter which cannot be pinned down by theory. This difficulty in defining dividends is precisely one reason why we regard the decomposition in preferences and dividends as a useful one. One can argue about a treatment of dividends in a given model but the role of preferences should be transparent. In this section, we follow the questionable approach and model dividends as claims to consumption. We do that not to defend this approach but to compare our model with the existing literature.

Essentially, we ask the following question: Suppose we have found set of preferences which produce a high enough Sharpe-ratio, are standard dividend processes capable of producing reasonable risk premia? Note that we can rewrite risk premia from equation (9) using the Sharpe-ratio as

$$r_{t+1}^{rp} = \text{SR } \eta_{r\epsilon} \sigma_{\epsilon}. \quad (15)$$

Figure 3: Leverage in the Mean-Standard Deviation Frontier



Note: Figure shows the effect of leverage for a given Sharpe-ratio of 0.27.

Recall that  $\eta_{r\epsilon}$  depends on the dividends of the asset. Lacking a better way to define equity in this setup, we follow the standard practice and define a claim to consumption as equity. First, consider for simplicity a 1-period claim to consumption. As mentioned earlier,  $\eta_{r\epsilon} = 1$  for this asset and  $\sigma_\epsilon = 0.56\%$ . Hence, the risk premium for the 1-period consumption claim, given a Sharpe-ratio of 0.27, is  $r_{t+1}^{rp} = 0.15\%$ . This number is too small by about a factor of 10 compared to the equity premium in the data. Hence consumption as dividends is not volatile enough even given the Sharpe-ratio from the data. To see how far the simple consumption claim is off, consider a levered claim to consumption:  $D_t = C_t^\xi$ , where  $\xi$  denotes the ‘leverage factor’. The risk premium is given by

$$r_{t+1}^{rp} = \text{SR} \xi \sigma_\epsilon. \quad (16)$$

Hence a leverage factor of 13.2 is required to generate a 2% equity premium. In terms of the mean-standard deviation diagram, leverage moves the position of the consumption claim on the CML as shown in Figure 3.

The argument above was based on a 1-period claim to consumption. Equity is a long run asset, however. It is well known that all term-premia are zero when log-consumption follows a random walk. So in this case, a long-run claim to consumption will carry the same risk premium as the 1-period claim. If consumption growth is autocorrelated, this is not true any longer. In post-war data, the autocorrelation of consumption growth is

0.16. Using this value, an infinite claim to consumption implies  $\eta_{rc} = 1.2$ . Hence the risk premium will be 20% higher than that of the one-period claim. Using the same argument as above, a  $\xi$  of about 11 is required to obtain a 2% risk premium for the infinite claim.

### 3.4 A Diagnosis

Risk premia depend on preferences and dividends. We decompose both effects using the Sharpe-ratio as a measure of the risk-return tradeoff implied by the preferences. Dividends determine the position of assets on the CML. We find that standard preferences and dividend processes fail on both counts. In other words, the equity premium puzzle is the sum of a *Sharpe-ratio puzzle* and *dividend puzzle*. In our opinion, both are completely separate issues and require separate research agendas. The Sharpe-ratio based on our lognormal model is furthermore useful as a very simple diagnostic tool for evaluating preferences in regard of their risk-return implications. It follows the spirit of the Hansen and Jagannathan (1991) bounds but is much easier to use since it depends only on the elasticity of the PSDF with respect to the consumption innovation. This parameter is easily calculated for any preference specification and does not require estimation of volatility bounds. A further advantage is that the analytical expression for the Sharpe-ratio allows for a transparent economic interpretation. The only substantive assumption needed is the conditional lognormality of consumption innovations.

### 3.5 Some Useful Properties

In this section we present some useful properties of the model. We consider possible changes in the location of the CML and show how other shocks affecting asset returns can be build into the model.

#### 3.5.1 Moving the CML

So far we implicitly assumed that the position of the CML is constant over time. However, there is evidence that the risk-free rate is changing over time as well evidence that the slope is not constant over time, see Campbell and Cochrane (1996) and the references therein. In our setup, it is easy to see what model ingredients cause what move in the CML. The risk-free rate is just the inverse of the expected PSDF. For the case of time-separable CRRA preferences any expected change in consumption will cause the risk-free rate to change. This could be due to serial correlation in log consumption. This will change the risk-free rate but will leave the Sharpe-ratio constant, hence the CML shifts up and down with predictable changes in consumption.

However, (14) shows what is needed to change the slope of the CML: either a time-varying elasticity of the PSDF or time-variation in the standard deviation of consumption



innovation. Campbell and Cochrane's (1996) nonlinear 'catching-up-with-the-Joneses' model produces a time-varying  $\eta_{m\epsilon,t}$  causing the CML to tilt around the risk-free rate.

### 3.5.2 Multiple Shocks

So far we assumed that the unexpected return of any asset is perfectly correlated with the shock to consumption, see equation (6). This amounts to assuming that log dividends are of the form  $d_t = \xi c_t$ . In other words, the model was a pure one-factor model. This implies that all assets are located *on* the CML. Figure 1 shows however, that long Gov't bonds are far below the CML. In other words the ratio of mean return to standard deviation is smaller. For a model which can produce this we need a second shock. For bonds this could for example be inflation risk. Hence we write the log return of an asset as

$$r_{t+1} = E_t r_{t+1} + \eta_{r\epsilon} \epsilon_{t+1} + \nu_{t+1} \quad (17)$$

$$= E_t r_{t+1} + w_{t+1}, \quad (18)$$

where  $w_{t+1} = \eta_{r\epsilon} \epsilon_{t+1} + \nu_{t+1}$ . We assume that  $\nu_t$  is normally distributed with zero mean and variance  $\sigma_\nu^2$ .<sup>10</sup> It is easily checked that the risk premia are unchanged since only covariance risk with the innovation of log consumption is priced. However, now the ratio of the conditional risk premium and conditional standard deviation is

$$\frac{r_{t+1}^{rp}}{\sigma_w} = \text{SR } \rho_{\epsilon w}, \quad (19)$$

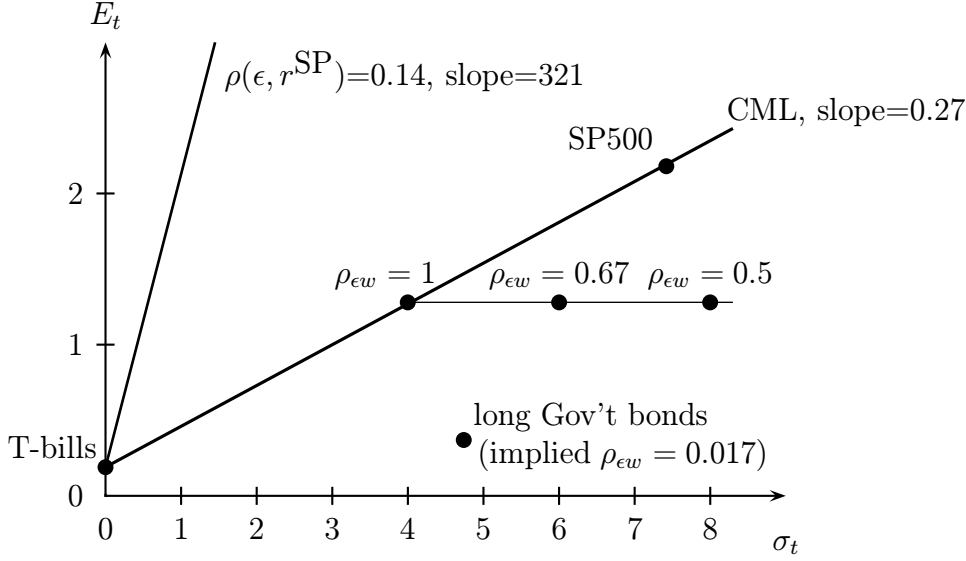
where  $\rho_{\epsilon w}$  is the correlation of  $\epsilon$  and  $w$ . Hence by increasing the variance of  $\nu$ , and therefore decreasing  $\rho_{\epsilon w}$ , the asset moves horizontally to the right away from the mean-standard deviation frontier, see 4. For long Government bonds the ratio of mean return to standard deviation is 0.038 implying  $\rho_{\epsilon w} = 0.17$ . Hence  $\sigma_\nu$  has to be about five times as large as  $\eta_{r\epsilon} \sigma_\epsilon$ . The unpriced (inflation) risk dominates the real risk.

Up to now, we implicitly assumed that the innovation in consumption  $\epsilon$  and returns of the SP500 index are perfectly correlated. This implies of course that the SP500 index lies on the CML. In the data this assumption is far from the facts. To get a rough idea, consider the following back-of-the-envelope calculation. Ignoring issues of conditonal versus unconditional moments and assuming log consumption follows a random walk, the correlation between consumption innovations and SP500 returns in post-war data is only 0.14. If we take this value as given, the true slope of the CML has to be  $0.27/0.14=1.92$ , see Figure 4. This in turn implies that a  $\eta_{m\epsilon} = -344$  is required to achieve a Sharpe-ratio of 1.92. This demonstrates clearly that a less than perfect correlation of the innovations in consumption and asset returns, which we find in the data, pose an additional challenge on top of the high risk premium for claims to consumption.

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<sup>10</sup>Again we could allow for time changing variance as well.

Figure 4: Multiple Factors in the Mean-Standard Deviation Frontier



Note: Figure shows the effect of a second shock to dividends for a given Sharpe-ratio of 0.27.

## 4 Alternative Preferences

In this section we evaluate some preference specification which have been studied in the literature using our Sharpe-ratio diagnostic tool. We consider the standard time-separable CRRA case, habit formation (Constantinides (1990)), ‘catching-up-with-the-Joneses’ (Abel (1990)), and nonexpected utility (Epstein and Zin (1991)) whether they are capable of producing a realistic risk-return tradeoff (i.e. a Sharpe-ratio of 0.27) for realistic parameter values. Recall that the only preference parameter which affects risk premia and the Sharpe-ratio is  $\eta_{m\epsilon}$ . Hence, even if different preferences look very different at first sight, they can imply *exactly* the same risk premia, given a consumption stream, if  $\eta_{m\epsilon}$  coincide. Another implication is that it is neither risk aversion nor intertemporal substitution which determines risk premia. As we will see, for certain preferences there will be a link between  $\eta_{m\epsilon}$  and risk aversion (e.g. CRRA and Epstein-Zin) and hence risk aversion determines risk premia. However, for other preferences such as habit formation,  $\eta_{m\epsilon}$  is linked to the elasticity of intertemporal substitution and not risk aversion. For those preferences risk aversion does not play a direct role for risk premia. Hence the deep parameter which controls risk premia for any general set of preferences is  $\eta_{m\epsilon}$ .

## 4.1 Time-separable CRRA

Consumer maximize the following utility function:

$$U_t = E_t \sum_{j=0}^{\infty} \beta^j \frac{C_{t+j}^{1-\gamma} - 1}{1-\gamma}. \quad (20)$$

The parameter  $\gamma$  measure RRA. Note that the inverse of  $\gamma$  equals the elasticity of intertemporal substitution (EIS):  $EIS = 1/\gamma$ . The log of the PSDF is given by

$$m_{t+1} = -\gamma \Delta c_{t+1} \quad (21)$$

and hence  $\eta_{m\epsilon} = -\gamma$ . As already calculated above, an extremely high RRA of about -50 is needed to match the Sharpe-ratio in the data. This corresponds to entering the volatility bounds of Hansen and Jagannathan (1991) for high RRA.

## 4.2 Habit Formation

Constantinides (1990) presents a preference specification in which utility not only depends on current consumption but also on lagged consumption (the habit):

$$U_t = E_t \sum_{j=0}^{\infty} \beta^j \frac{(C_{t+j} - \theta C_{t+j-1})^{1-\gamma} - 1}{1-\gamma}. \quad (22)$$

Constantinides (1990) proposes a slightly more general process for the habit component. However, this specification is general enough to make our point. The PSDF can be written as

$$M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{(1 - \theta C_t / C_{t+1})^{-\gamma} - \beta \theta E_{t+1} \left[ \left( \frac{C_{t+2}}{C_{t+1}} - \theta \right)^{-\gamma} \right]}{(1 - \theta C_{t-1} / C_t)^{-\gamma} - \beta \theta E_t \left[ \left( \frac{C_{t+1}}{C_t} - \theta \right)^{-\gamma} \right]} \quad (23)$$

Note that (23) reduces to (21) for  $\theta = 0$ . Since conditional expectations have to be computed, we assume at this point that log consumption is a random walk with drift:  $c_{t+1} = g + c_t + \epsilon_{t+1}$ . Straightforward algebra shows that the elasticity  $\eta_{m\epsilon}$  equals

$$\eta_{m\epsilon} = -\frac{\gamma}{1 - \bar{x}} \frac{1 + \beta \theta \bar{x} e^{-g\gamma}}{1 - \beta \theta e^{-g\gamma}} \quad (24)$$

where  $\bar{x} = \bar{X}/\bar{C} = \theta e^{-g}$  is the steady state proportion of the habit of total consumption.  $\eta_{m\epsilon}$  is the sum of two parts. The first term in (24) represents the effect that *effective* consumption is consumption expenditures minus the habit. Hence the effect of the parameter  $\gamma$  in (22) is dividend by the proportion of consumption net of the habit of total consumption. The second term is due to the fact that an increase in consumption today has an effect on marginal utility tomorrow. Note that  $\eta_{m\epsilon}$  measures the curvature of the utility function (22) with respect to consumption. As Constantinides (1990)

Table 2: RRA vs.  $\eta_{m\epsilon}$ : Habit Formation  
parameters:  $\beta = 0.95, g = 0.44\%, \gamma = 1$

$\theta$	RRA	$\eta_{m\epsilon}$	$-\gamma/(1 - \bar{x})$
0.1	1.005	-1.238	-1.111
0.3	1.011	-2.159	-1.426
0.5	1.046	-4.667	-1.991
0.7	1.106	-14.27	-3.300
0.8	1.182	-32.36	-4.914
0.9	1.410	-113.99	-9.620
0.95	1.863	-336.56	-18.468
0.99	5.427	-2106.52	-69.709

Note: Table shows RRA using (25) and  $\eta_{m\epsilon}$  using (24) for different values of  $\theta$ . Parameters  $\beta$  and  $g$  are calibrated from postwar US data.  $\gamma$  is set to unity.

stresses risk aversion measures the reluctance of consumers to accept atemporal gambles in *wealth*. He shows that risk aversion is decoupled from the parameter  $\gamma$  as follows:

$$\text{RRA} = \frac{\gamma}{1 - \bar{x} \frac{e^{-\gamma g - \beta \gamma}}{e^{-\gamma g - \beta \theta}}}. \quad (25)$$

Table 2 shows RRA and  $\eta_{m\epsilon}$  for different values of  $\theta$ . The curvature in the period utility function  $\gamma$  is set to unity. Increasing  $\theta$  increases  $\eta_{m\epsilon}$  much more than RRA. The reason is that habit formation consumers are not that averse to gambles in wealth because they can slowly adjust consumption in response to a decrease in wealth. In contrast, they are very reluctant to suddenly adjust consumption since this will require them to consume more in the future as well. This distinction has been discussed in quite some depth by Boldrin, Christiano and Fisher (1995). Recall that a  $\eta_{m\epsilon}$  of around 50 is needed to obtain the point estimate of the Sharpe-ratio of 0.27. The table shows that this can be achieved by setting  $\theta$  to about 0.82 with  $\gamma = 1$ . However this implies that the proportion of the habit is around 82% of total consumption. Alternatively, one can increase  $\gamma$  and select a lower  $\theta$ . Note that risk aversion does not increase to unrealistic values in either case.

However, as we will argue later a high  $\eta_{m\epsilon}$  is equally unrealistic on theoretical grounds as a high RRA. Recall that  $\eta_{m\epsilon}$  measures the reluctance to change consumption between periods. A high  $\eta_{m\epsilon}$  implies that consumers are extremely unwilling to adjust consumption. In the Lucas (1978) model this has no consequences because agents cannot transfer consumption at all from one period to another. However, if there are any smoothing channels which enable consumers to smooth consumption, a high  $\eta_{m\epsilon}$  implies that they

want to use these channels and create a smooth consumption path. In the next section we will demonstrate that consumption will be far too smooth in models with even moderate smoothing channels if the EIS is very low. This raises serious doubt whether models which require a high  $\eta_{m\epsilon}$  via a low EIS are really a solution for the equity premium puzzle.

### 4.3 ‘Catching-up-with-the-Joneses’

Abel (1990) presents a simplified model of habit persistence where individual utility depends on individual consumption as well as on aggregate consumption. It can easily be shown that the PSDF of the individual consumer is similar to (23) with all the terms regarding the effect on marginal utility in  $t + 1$  are ignored, i.e. the terms in square brackets in (23) equal zero. In other words, the individual does not take the effect on marginal utility of changing consumption into account. With this preference specification, the consumption externality which was present in habit formation preferences disappears. Therefore we find that  $RRA = -\eta_{m\epsilon} = \gamma/(1 - \bar{x})$ . Hence, creating a high Sharpe-ratio is only possible at the cost of a high RRA and low EIS. Table 2 reports values for  $\eta_{m\epsilon} = -\gamma/(1 - \bar{x})$  for different  $\theta$ .

### 4.4 ‘Catching-up-with-the-Joneses’ when habit depends on *current* consumption

Preferences (22) assume that habit depends on lagged consumption. In a recent paper, Campbell and Cochrane (1996) allow habit to depend on current consumption. As we will show below this drives again a wedge between RRA and the EIS, and the Sharpe-ratio depends on the EIS. Campbell and Cochrane set up the model as follows (see their paper for details). Consumers maximize

$$U_t = E_t \sum_{j=0}^{\infty} \beta^j \frac{(C_{t+j} - X_{t+j})^{1-\gamma} - 1}{1 - \gamma}. \quad (26)$$

The habit  $X_t$  is allowed to depend on current consumption. Instead of positing a process for  $X_t$ , Campbell and Cochrane choose to work with the surplus ratio  $S_t = (C_t - X_t)/X_t$ . Since the habit is assumed to be external, consumers take  $S_{t+1}$  as given when computing their PSDF:

$$M_{t+1} = \beta \left( \frac{S_{t+1} C_{t+1}}{S_t C_t} \right)^{-\gamma}. \quad (27)$$

Campbell and Cochrane show that RRA is given by  $\gamma/S_t$ . Note that since  $S_t$  is varying over time, RRA is moving as well. In bad times (i.e.  $S_t$  is small), the consumers will be more risk averse. To close the model, the process for linking  $S_t$  to consumption has to be specified. Campbell and Cochrane assume for  $s_t = \log S_t$ :

$$s_t = (1 - \phi)\bar{s} + \phi s_{t-1} + \lambda(s_{t-1}) \epsilon_t.$$

Given these assumption it is straightforward to compute  $\eta_{m\epsilon,t}$ :

$$\eta_{m\epsilon,t} = -\frac{1}{\text{EIS}_t} = -\gamma(1 + \lambda(s_t)). \quad (28)$$

Several things are worth noting. First,  $\eta_{m\epsilon,t}$  and hence asset prices depend on the EIS as in the internal habit case. Second, risk aversion does not directly affect risk premia. Third, the Sharpe-ratio and therefore risk premia are time-varying only if the function  $\lambda(\cdot)$  is nonconstant. Hence, the model can imply a time-varying RRA with constant risk premia. Campbell and Cochrane stress the importance of time-variation in the Sharpe-ratio which can only be achieved when the EIS is changing over time.

## 4.5 Non-expected Utility

Building on Kreps and Porteus (1978), Epstein and Zin (1991) have formulated a preference specification which allows the distinction between RRA and the elasticity of intertemporal substitution (EIS). The objective function is defined recursively as

$$U_t = \left[ (1 - \beta)C_t^{1-\sigma^{-1}} + \beta \left( E_t U_{t+1}^{1-\gamma} \right)^{\frac{1-\sigma^{-1}}{1-\gamma}} \right]^{1/(1-\sigma^{-1})} \quad (29)$$

The notation is from Campbell (1993).  $\gamma$  is the coefficient of RRA and  $\sigma$  is the EIS. See Epstein and Zin (1991), Giovannini and Weil (1989) and Campbell (1993) for a more detailed description of these preferences.

It is straightforward to calculate the preference-based stochastic discount factor for pricing assets  $M_{t+1}$  for these preferences, see equation (46) in appendix A.3. However,  $M_{t+1}$  involves “unobservables” such as  $U_{t+1}$ . In order to make the asset pricing equation econometrically more useful, Epstein and Zin (1989,1991) and essentially the entire literature following it, made the additional assumption, that the agent is representative, that he owns assets, the returns to which he can either consume or reinvest, and that there is no other source of income. Using these obviously highly restrictive assumptions, Epstein and Zin (1991) have shown, that the PSDF can be rewritten as

$$M_{t+1} = \beta^{\frac{1-\gamma}{1-\sigma^{-1}}} \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1-\gamma}{\sigma^{-1}}} R_{m,t+1}^{\frac{\sigma^{-1}-\gamma}{1-\sigma^{-1}}}, \quad (30)$$

where  $R_m$  is the gross return on invested wealth (i.e. the market portfolio). Since our aim is to interpret the findings of the literature, we follow its footsteps and proceed with this equation rather than equation (46).

In logs the PSDF can then be rewritten as

$$m_{t+1} = \frac{1-\gamma}{1-\sigma^{-1}} \log \beta - \frac{1-\gamma}{\sigma^{-1}-1} \left( \Delta c_{t+1} + \frac{1-\sigma\gamma}{1-\gamma} r_{m,t+1} \right) \quad (31)$$

It seems that an EIS close unity can generate large  $\eta_{me}$  irrespective of RRA. However, as shown by Campbell (1993) the model implies a relationship between unexpected consumption and unexpected return on the market portfolio:

$$c_{t+1} - E_t c_{t+1} = r_{m,t+1} - E_t r_{m,t+1} + (1 - \sigma)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{m,t+1+j}, \quad (32)$$

where  $\rho$  is a parameter related to the consumption-wealth ratio and is close to unity. Substituting out the market return in (31), we get

$$m_{t+1} = E_t m_{t+1} - \gamma \epsilon_{t+1} - (1 - \sigma\gamma) \lambda_{m,t+1}, \quad (33)$$

where  $\lambda_{m,t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{m,t+1+j}$  represents the ‘news’ of future expected returns of the market portfolio between period  $t$  and  $t+1$ . Equation (33) tells us that an EIS close unity does *not* imply a high  $\eta_{me}$ . The impact of the consumption innovation on the PSDF depends only on RRA, as in the CRRA case. The EIS only affects the PSDF through changes in expected returns of the market portfolio. If returns are i.i.d. or if  $\gamma = 1/\sigma$  we obtain the same PSDF as in the time-separable CRRA case (see also Campbell (1993) for a more extensive discussion). The effect of  $\lambda_m$  in (33) is likely to be fairly small since returns are difficult to forecast. Therefore, the Epstein and Zin preferences require a high risk aversion to generate a high Sharpe-ratio. In this regard they are only a small improvement over the standard time-separable CRRA preferences. Of course, what the Epstein-Zin preferences do accomplish is to allow for high risk aversion without simultaneously implying a low EIS.

## 4.6 A Summary and Outlook

Before turning to the issue of consumption smoothing, we want to summarize the important properties of the different preferences considered above. In order to generate a high Sharpe-ratio, the standard time-separable CRRA preference specification requires a high coefficient of RRA of around 50 which implies a low EIS of about 0.02. Epstein-Zin preferences disentangle RRA from intertemporal substitution. RRA of around 50 is still required to produce a high Sharpe-ratio but the EIS is not restricted. It is an open issue whether such high RRA is reasonable. It is obviously very hard to estimate risk aversion of a representative agent directly. Some experimental evidence suggests that risk aversion is generally not that high. As an illustration of the implications of RRA of 50, consider an agent who is faced with a 50-50 gamble of gaining or losing 10% of her total wealth. An agent with RRA of unity would be willing to pay 0.5% of her wealth to avoid that gamble, an agent with RRA of 50 would be willing to pay 8.7% of her wealth. In light of these numbers, we regard such high RRA as implausible.

This line of argument motivated Constantinides (1990) to study the influence of habit formation on asset prices. With habit formation risk premia can be high with low risk

aversion. However, the EIS is required to be around 0.02, just as in the standard CRRA case. In the next section we will show that such a low EIS implies that consumers prefer an extremely smooth consumption path. In an exchange economy this is of no consequence for the resulting consumption path because agents have no opportunity to transfer consumption from one period to the next. However, once there are any smoothing channels available (such as a storage technology), agents with a low EIS will use these possibilities to create a less volatile consumption path. This in turn will lower the Sharpe-ratio. In other words, preferences with a low EIS imply low risk premia in economies with consumption smoothing channels.

## 5 Consumption Smoothing

The Lucas (1978) model has been the basis for consumption-based asset pricing. It has been a useful easy-to-use tool to compute risk premia as a function on their covariance risk with consumption. However, the simplicity of the model has also led to an extreme one-sided view, namely how consumption, via preferences, affects asset prices. The basis for this approach is a pure exchange economy. Agents are forced to consume their (exogenously) given endowment. They do not have access to any storage technology for the consumption good. Through this modeling trick the consumption process can be viewed as being exogenous to the model. Preferences of consumers determine then the capital market line as shown above. The advantage of this approach is that a realistic consumption process can be assumed from the outset. The underlying presumption is that consumption is determined outside the model and can hence be taken to be exogenous.

While this approach is useful as a first test for preferences, a more complete model should include consumption as a real choice variable. The danger of the search for the ‘right’ preferences using exchange economies is that it is unlikely that these preferences generate the assumed consumption process. In this sense, the exchange economy can serve as a necessary but not sufficient condition in the search for preferences. A more complete model should always be kept in mind when one is writing down a pure exchange economy. In this section, we make this argument more precise using a simple extension to the exchange economy where a storage technology allows consumers to transfer goods between periods. In other words, the consumption good can be converted into a storable capital good. The underlying shock is assumed to affect the productivity of the firm (or the trees in the Lucas economy). The properties of the stochastic process of productivity are taken from the standard practice in macroeconomics to match the Solow residual. The following equations are added to the exchange economy. Consumers face an intertemporal budget constraint:

$$C_t + K_t = Y_t + (1 - \delta)K_{t-1}, \quad (34)$$



where  $K_t$  denotes the stock of capital,  $Y_t$  denotes output and  $\delta$  is a depreciation factor. Output itself is produced using capital as an input:

$$Y_t = Z_t K_{t-1}^\rho. \quad (35)$$

$Z_t$  is a productivity parameter.  $z_t = \log Z_t$  is assumed to follow the stochastic process

$$z_t = \psi z_{t-1} + \zeta_t, \quad \zeta_t \sim \text{i.i.d. } N(0, \sigma_\zeta^2). \quad (36)$$

The variance of  $\zeta$  can be measured using the Solow residual, Hansen (1985) uses  $\sigma_\zeta = 0.763\%$ , which we will also use. Note that this number is slightly higher than  $\sigma_\epsilon$ . Usually it is assumed that shocks are highly persistent, we use  $\psi = 0.95$ . The depreciation rate is set to  $\delta = 0.025$ . The model is solved using the log-linear technique described e.g. by Campbell (1994) and Uhlig (1995).

The key difference to the exchange economy is how consumption is determined. In this model agents can save part of a shock  $\zeta_t$ . Let  $\eta_{c\zeta}$  denote the elasticity of consumption with respect to  $\zeta$ . In an exchange economy this elasticity is by assumption equal to unity. In contrast,  $\eta_{c\zeta}$  depends on the preferences of agents in the economy with storage, in other words it is an endogenous variable. The innovation to consumption can be written as  $\epsilon_t = \eta_{c\zeta} \zeta_t$ . Hence we can write the Sharpe-ratio

$$\text{SR} = -\eta_{m\epsilon} \sigma_\epsilon \quad (37)$$

$$= -\eta_{m\epsilon} \eta_{c\zeta} \sigma_\zeta. \quad (38)$$

The key insight is that changing the preference parameter  $\eta_{m\epsilon}$  will affect the consumption path and therefore  $\eta_{c\zeta}$ . To be more concrete, take preferences where  $-\eta_{m\epsilon} = 1/\text{EIS}$  such as time-separable CRRA or habit formation. Lowering the EIS will make consumers more reluctant to change consumption over time. Hence they will save a larger proportion of a positive productivity shock which implies a lower  $\eta_{m\zeta}$ . The effect of a lower EIS increases the Sharpe-ratio while a lower  $\eta_{m\zeta}$  decreases it. The net effect is ambiguous. In other word, preferences with a low EIS tend to generate high Sharpe-ratios in exchange economies but fail to do so in economies with savings.

## 5.1 Exchange Economy vs. Savings Economy

Table 3 compares the asset pricing implications in an exchange economy and an economy with savings as described above. The table reports the Sharpe-ratio, the risk premium of an infinite claim to consumption as well as the elasticity of consumption with respect to an exogenous shock,  $\eta_{c\zeta}$ . In an exchange economy  $\eta_{c\zeta}$  is always unity by assumption. In an economy where consumers can save, they will generally save a fraction of a positive shock to productivity. How much they want to save depends on the the EIS implied by their preferences. We consider five preference specifications in Table 3. The first three

Table 3: Exchange Economy vs. Economy with Savings

data: SR=0.27, EqPrem=2%

Preference Parameters		Exchange Economy	Economy with Savings
$\gamma = 1$	SR	0.0056	0.0013
	$\eta_{c\zeta}$	1.0000	0.2264
	CPrem	0.0031	0.0005
$\gamma = 10$	SR	0.0560	0.0171
	$\eta_{c\zeta}$	1.0000	0.2368
	CPrem	0.0314	0.0189
$\gamma = 50$	SR	0.2800	0.0695
	$\eta_{c\zeta}$	1.0000	0.1931
	CPrem	0.1568	0.1325
$\gamma = 2.1$ $\theta = 0.5$ ( $\eta_{m\epsilon} = -10$ , RRA=2.2)	SR	0.0560	0.0069
	$\eta_{c\zeta}$	1.0000	0.0947
	CPrem	0.0314	0.0012
$\gamma = 1.6$ $\theta = 0.8$ ( $\eta_{m\epsilon} = -50$ , RRA=1.9)	SR	0.2800	0.0108
	$\eta_{c\zeta}$	1.0000	0.0296
	CPrem	0.1568	0.0067

Note: Table shows asset prices for different preferences in an exchange economy and an economy with savings. SR is the Sharpe-ratio and  $\eta_{c\zeta}$  denotes the elasticity of consumption with respect to an exogenous shock. CPrem is the risk premium (in %) on an infinite stream to a consumption claim. The log-consumption process in the exchange economy is  $c_{t+1} = 0.0044 + c_t + \zeta_{t+1}$ ,  $\zeta_{t+1} \sim N(0, 0.0056^2)$ . The parameters in the savings economy are as in the text.

are standard time-separable CRRA preferences with different RRA. The other two cases are habit formation preferences as described in Section 4.2. The parameters of the habit formation preferences are chosen to match the  $\eta_{m\epsilon}$  of the CRRA preferences for  $\gamma = 10$  and 50 respectively. Recall that for habit formation preferences the EIS is minus the inverse of  $\eta_{m\epsilon}$ .

Consider first the CRRA preferences with  $\gamma = 1$ . Using (14) the Sharpe-ratio in the exchange economy is equal to the standard deviation of the consumption innovation. As already discussed in Section 3.2 this value is too small by a factor of 50. Increasing RRA in the exchange economy proportionally increases the Sharpe-ratio. Hence a RRA of 50 delivers a Sharpe-ratio which matches the data. Note that the habit preferences imply the exact same Sharpe-ratios as the CRRA preferences for matching  $\eta_{m\epsilon}$ . To demonstrate that risk aversion does not control asset prices, note that the preference parameter in row 4 imply a higher RRA than those in row five. The Sharpe-ratio is larger for the *low* risk aversion case. The risk premium on the infinitely lived claim to consumption is far too small for any of the preferences (see the discussion in Section 3.3).

The picture is quite different in the economy with savings possibilities. For  $\gamma = 1$  agents consume only 22.64% of a shock and hence save 77.36% of the effect of the shock. This reduces the Sharpe-ratio to 0.0013, less than one fourth of the value in the exchange economy. Increasing the risk aversion in the savings economy does not change the reaction of consumption after a shock very much. Consumers with CRRA preferences save around 80% of the shock. The effect of savings for the habit formation consumer is even more dramatic. For  $\gamma = 2.1$  and  $\theta = 0.5$  only 10% of the shock is saved dramatically reducing the Sharpe-ratio in comparison to the exchange economy. The reason for smaller consumption response compared to the CRRA preferences with a corresponding EIS of 1/10 is the effect of today's consumption on tomorrow's marginal utility. Habit formation consumers prefer a smoother adjustment of consumption. A stronger habit ( $\theta = 0.8, \gamma = 1.6$ ) leads to an even slower adjustment. Note that the risk premia on the infinite consumption stream are all small in the savings economy, largely due to the smaller Sharpe-ratio.

The bottom line of this section is that preferences which are based on a low EIS may produce high Sharpe-ratios in exchange economies but they will fail to do so once agents are allowed to save from period to period. In this section we computed asset prices in an economy with only one channel through which consumption can be smoothed. Most macroeconomic models include a second channel, namely labor input. Once labor is added to the production function and leisure to the preferences, agents can vary their labor input in response to shocks. For more detailed results of such a model see Lettau and Uhlig (1996). In general the response of consumption to a shock is much smaller and hence the Sharpe-ratios are decreasing as well. To give a representative number, a consumer in the Hansen (1985) model with variable labor input will consume only 1.4%

of a shock for a RRA of 50. So the more smoothing channels are available to agent, the smaller the Sharpe-ratio. The exchange economy is an extreme point in this spectrum delivering the largest possible Sharpe-ratio.

## 6 Idiosyncratic Shocks

In light of the rather bleak outlook for the complete markets model considered above, researchers have expanded the standard model to allow for idiosyncratic risk for the individual consumer. Constantinides and Duffy (1996), Den Haan (1996) and Heaton and Lucas (1996) have studied versions of these models as a possible avenue to explain high risk premia. In this section we will use the Sharpe-ratio to evaluate the potential of models with idiosyncratic shocks. We will show that ‘simple’ versions of these models will probably not be successful in increasing the Sharpe-ratio in an economy. Instead it takes more elaborate setups as in Constantinides and Duffy (1996) which operate on second moments of the cross-sectional distribution of agents.

Suppose log consumption of consumer  $i$  is given by

$$c_{t+1}^i = E_t c_{t+1}^i + \epsilon_{t+1} + \epsilon_{t+1}^i, \quad (39)$$

where  $\epsilon_{t+1}$  represents the aggregate shock common to each consumer and  $\epsilon_{t+1}^i$  is the idiosyncratic shock of consumer  $i$ . We assume that  $\epsilon_{t+1}$  and  $\epsilon_{t+1}^i$  are independently normally distributed with zero mean and variance  $\sigma_\epsilon^2$  and  $\sigma_{\epsilon^i}^2$ , respectively. The log IMRS of consumer  $i$  is then

$$m_{t+1}^i = E_t m_{t+1}^i + \eta_{m\epsilon}(\epsilon_{t+1} + \epsilon_{t+1}^i). \quad (40)$$

A quick calculation using (14) suggests that the Sharpe-ratio becomes

$$SR = -\eta_{m\epsilon} \sqrt{\sigma_\epsilon^2 + \sigma_{\epsilon^i}^2}. \quad (41)$$

For  $\eta_{m\epsilon} = -5$  and  $\sigma_\epsilon = 0.56\%$ , the standard deviation of the idiosyncratic component has to be  $\sigma_{\epsilon^i}$  has to be  $5.4\%$  to create a Sharpe-ratio of 0.27. Hence the idiosyncratic consumption has to be about ten times as large as the aggregate consumption risk.

However, a closer look shows that equation (41) is misleading. Recall the definition of the Sharpe-ratio in (10). In constructing the asset with a conditional correlation of -1 with the IMRS we needed an asset which is perfectly negatively correlated with the IMRS. In the case without idiosyncratic risk, one such asset is a one-period consumption claim. However, in a model with idiosyncratic consumption risk, the individual IMRS enters the definition of the Sharpe-ratio. Hence an asset with  $\rho_t(M_{t+1}^i, R_{t+1}) = -1$  has to have a return which is perfectly negatively correlated with *individual* consumption. To see this first note that  $\rho(e^x, e^y) \approx \rho(x, y)$  for normal  $x$  and  $y$  and small covariance.

<sup>11</sup> Consider some one-period asset with dividend  $d_{t+1}$ , then

$$-\rho_t(m_{t+1}, r_{t+1}) = \rho_t(\epsilon_{t+1} + \epsilon_{t+1}^i, d_{t+1}). \quad (42)$$

This shows that  $d_{t+1}$  has to be perfectly correlated with the aggregate *and* idiosyncratic consumption shocks to achieve a perfect correlation. One such asset would a claim to *individual* consumption. The existence of such assets is questionable because of the very nature of idiosyncratic risk. If insurance market for these types of risk do not exist, how can there be an asset whose dividend depend on these risks? If these assets existed, then the agent would use these assets to insure herself against these risks.

If there are no assets with dividends depending on the idiosyncratic consumption risk, then it is straightforward to show the the Sharpe-ratio is again  $SR = -\eta_{m\epsilon}\sigma_\epsilon$ , the same as in the model without idiosyncratic risk.

This argument shows that just adding some uninsurable income risk to the standard complete markets model will not substantially increase risk premia. Den Haan (1996) and Heaton and Lucas (1996) have studied models in which subsets of agents are subject to uninsurable shocks. Heaton and Lucas (1996) report somewhat higher risk premia in their economy with two groups of agents, each of which is subject to an idiosyncratic shock. The reason for higher risk premia in such a model is that half of the population is subject to a shock. If many agents are affected by a shock their reaction will have an effect on the entire market. This causes a correlation between a shock and aggregate dividends which in turn increases the Sharpe-ratio, as shown in (42). Note that this argument depends on the market power of the agents which are subject to a common shock. Den Haan (1996) has shown that it is much harder to increase risk premia in models in which each individual agent is subject to a shock. Since one agent cannot affect the aggregate market there is no correlation with an idiosyncratic shock. Hence the Sharpe-ratio will remain low.

Constantinides and Duffy (1996) present a more elaborate model in which a the conditional variance of the cross-sectional distribution enters pricing kernel. This variance is varying over time and can be correlated with aggregate dividends. This correlation can increase the Sharpe-ratio. In order to allow for this channel working through second moments in our setup, we would have to relax the assumption of constant variances of the shocks. What the argument in this section shows, however, is that simple models including idiosyncratic shocks will not increase the risk-return tradeoff substantially. Only more complicated models working through second moments have the potential to increase the Sharpe-ratio in an economy.

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<sup>11</sup>The exact expression is  $\rho(e^x, e^y) = (e^{\sigma_{xy}} - 1)/(\sigma_x \sigma_y)$ .

## 7 Conclusion

In this paper we showed how the role of preferences and dividends concerning risk premia can be separated. Preferences determine the position of the capital market line while dividends determine the position of an asset on or below the capital market line. We used the Sharpe-ratio as a general measure of the risk - return tradeoff implied by preferences. The only preference parameter affecting the Sharpe-ratio is the elasticity of a preference-based stochastic discount factor for pricing assets with respect to the consumption innovation. We relate this preference discount factor elasticity to more standard preference parameters such as relative risk aversion or the elasticity of intertemporal substitution. For some preferences the preference factor elasticity is related to relative risk aversion, for other preferences it is related to the elasticity of intertemporal substitution. Independently of the particular preferences, a preference factor elasticity of about -50 is required to match the Sharpe-ratio in post-war data. For preferences often considered in the literature, this requires either a high relative risk aversion of 50 (for Epstein-Zin preferences), a low elasticity of intertemporal substitution of 0.02 (for habit formation) or both (for time-separable constant relative risk aversion). While a relative risk aversion of 50 is often regarded as unplausible, we argue that a low elasticity of intertemporal substitution of 0.02 is also hard to accept because agents will smooth consumption extremely if at all possible. Hence we conclude that none of the preferences used in the literature provides a satisfying solution for the Sharpe-ratio puzzle. We also show that simple models with incomplete markets such as uninsurable income risk may not help to increase the Sharpe-ratio because dividends have to be correlated not only with aggregate consumption but also with individual (uninsurable) consumption.

## A Appendix

In this appendix, we show that our generalized form of preferences includes all the preference specifications analyzed in this paper, and we derive the asset pricing formula. We follow the dating convention that anything dated  $t$  is measurable with respect to the information at date  $t$ , but, typically, not earlier.

First, we repeat the assumptions made in the main text, allowing in slight generalization for a vector of consumption goods. Let  $(S_t)_t$  be a Markov process on some state space and let consumption  $C_t \in \mathbf{R}^{n_C}$  be determined by a function  $C_t = C(S_t)$  of the state  $S_t$ . Let  $Z_t \in \mathbf{R}^{n_Z}$  denote “summary” information about past consumption, evolving according to a summarizer function  $g$ :

$$Z_t = g(C_t, Z_{t-1})$$

Let  $V(S_t, Z_{t-1}) \in \mathbf{R}$  be the “value function” of the remaining stream of consumption as of date  $t$ , evolving according to

$$V(S, Z) = f(C(S), Z, E[h(V(S', g(C(S), Z))) \mid S], S) \quad (43)$$

where  $f(C, Z, E, S) \in \mathbf{R}$  is an aggregator function and  $h(V) \in \mathbf{R}$  is a (typically monotone) transformation of  $V$ . The agent prefers higher values of  $V(\cdot, \cdot)$  to lower values. Given everything else, it is not clear at all, that a function  $V(\cdot, \cdot)$  satisfying the functional equation (43) exists: surely, any proof will require some additional restrictions. In fact, proving existence for essentially a special case is much of the work in Epstein and Zin (1989). For the purpose of the paper here we do not intend to generalize their analysis. Instead, we simply want to think of (43) as a general form in which several, well-known and well-behaved specifications can be stated, and for which a first-order approach is justified, and which may include some further interesting and more general cases as well.

### A.1 Special cases

We show now that the preference specifications in the main text fit the description. We keep the description of the consumption process as above: clearly, a random walk for consumption is contained as a special case.

**Separable CRRA:** In the “standard case” of a separable CRRA, just let

$$\begin{aligned} n_C &= 1 \\ n_Z &= 0 \\ g(C, Z) &\equiv 0 \\ f(C, Z, E, S) &= \frac{C^{1-\gamma} - 1}{1-\gamma} + \beta E \\ h(V) &= V \end{aligned}$$

**Habit Formation:** For the habit formation specification used in the text, let

$$\begin{aligned}
n_C &= 1 \\
n_Z &= 1 \\
g(C, Z) &= C \\
f(C, Z, E, S) &= \frac{(C - \theta Z)^{1-\gamma} - 1}{1 - \gamma} + \beta E \\
h(V) &= V
\end{aligned}$$

**Catching up with the Joneses:** Let  $\bar{C} = \bar{C}(S)$  be aggregate consumption as a function of the current state  $s$ . Let

$$\begin{aligned}
n_C &= 1 \\
n_Z &= 0 \\
g(C, Z) &= 0 \\
f(C, Z, E, S) &= \frac{(C - \theta \bar{C}(S))^{1-\gamma} - 1}{1 - \gamma} + \beta E \\
h(V) &= V
\end{aligned}$$

**Non-expected utility:** Using the notation in the main text, let

$$\begin{aligned}
n_C &= 1 \\
n_Z &= 0 \\
g(C, Z) &= 0 \\
f(C, Z, E, S) &= \left( (1 - \beta)C^{1-\sigma^{-1}} + \beta E^{\frac{1-\sigma^{-1}}{1-\gamma}} \right)^{\frac{1}{1-\sigma^{-1}}} \\
h(V) &= V^{1-\gamma}
\end{aligned}$$

so that  $V(S_t, Z_{t-1}) = U_t$ .

None of the examples includes several consumption goods such as leisure, but it is easy to think of further examples, and is clear that the general description allows for them.

## A.2 The Lucas asset pricing equation

To derive the Lucas asset pricing equation, we perform the following mind experiment. Suppose there is some asset, which can be purchased for the consumption bundle  $\tilde{C} \in \mathbf{R}^{n_C}$  at date  $t$  and which pays the random consumption bundle  $R = R_{t+1} \in \mathbf{R}^{n_C}$  at date  $t + 1$ .

Imagine investing a marginal amount in this asset by reducing consumption at date  $t$  by some amount  $\tilde{C}\epsilon$ , where  $\epsilon \in \mathbf{R}$  and  $|\epsilon|$  is small, and using the proceeds to increase



consumption at date  $t + 1$  by the amount  $R\epsilon$ . If the agent had the possibility to trade in this asset when solving for his optimal consumption plan, and if the agent was not restricted in the direction of the trade in that asset, then this mind experiment should not result in an increase in value for the agent. Taking  $\epsilon \rightarrow 0$ , we arrive at a marginal condition, which needs to be satisfied: we assume that we are allowed to differentiate and to exchange expectations and differentiation, whenever needed.

To perform the analysis, let  $v(\epsilon) = V(s, z; \epsilon)$  denote the value  $V(\cdot, \cdot)$  for some  $\epsilon$ , in which consumption is altered in the manner described above. Written explicitly, we get

$$\begin{aligned} v(\epsilon) &= f\left(C(S) - \tilde{C}\epsilon, Z, \dots\right. \\ &\quad E\left[h\left(f\left(C(S') + R\epsilon, g(C(S) - \tilde{C}\epsilon, Z), \dots\right.\right. \\ &\quad \quad \left.\left.\left.E\left[h\left(V\left(S'', g\left(C(S') + R\epsilon, g(C(S) - \tilde{C}\epsilon, Z)\right)\right)\right] \mid S'\right], S'\right)\right) \mid S\right], S) \end{aligned}$$

To abbreviate the notation, let

$$\begin{aligned} E_t[\cdot] &= E[\cdot \mid S] \\ V_t &= V(S, Z) \\ f_t &= f(C(S), Z, E[V(S', g(C(S), Z)) \mid S], S) \\ g_t &= g(C(S), Z) \\ h_t &= h(V_t) \end{aligned}$$

and likewise for  $E_{t+1}$ ,  $V_{t+1}$ ,  $V_{t+2}$ ,  $f_{t+1}$ ,  $g_{t+1}$ ,  $h_{t+1}$ ,  $h_{t+2}$  and their partial derivatives,  $f_{t,1}$ ,  $f_{t+1,1}$ , etc.. Calculating  $v'(0) = 0$ , one gets

$$\begin{aligned} 0 &= -f_{t,1}\tilde{C} \\ &\quad + f_{t,3}E_t\left[h'_{t+1}\left(f_{t+1,1}R - f_{t+1,2}g_{t,1}\tilde{C} + \right.\right. \\ &\quad \quad \left.\left.f_{t+1,3}E_{t+1}\left[h'_{t+2}V_{t+2,2}(g_{t+1,1}R - g_{t+1,2}g_{t,1}\tilde{C})\right]\right)\right] \end{aligned}$$

Collect terms: let

$$\begin{aligned} \Psi_t &= f_{t,1} + f_{t,3}E_t\left[h'_{t+1}\left(f_{t+1,2}g_{t,1} + f_{t+1,3}E_{t+1}\left[h'_{t+2}V_{t+2,2}\right]g_{t+1,2}g_{t,1}\right)\right] \\ \Phi_{t+1} &= f_{t,3}h'_{t+1}\left(f_{t+1,1} + f_{t+1,3}E_{t+1}\left[h'_{t+2}V_{t+2,2}\right]g_{t+1,1}\right) \end{aligned}$$

Note, that  $\Psi_t\tilde{C} \in \mathbf{R}$ . Let

$$M_{t+1} = \frac{\Phi_{t+1}}{\Psi_t\tilde{C}} \quad (44)$$

It follows that

$$1 = E_t[M_{t+1}R_{t+1}] \quad (45)$$

as claimed, where we have used  $R_{t+1}$  instead of  $R$  again. The important point here is, that  $M_{t+1}$  depends on the asset only via the “price”  $\tilde{C}$ . Note, that if spot markets

are complete, then there is no loss in generality in assuming that  $\tilde{C}$  is the same vector independent of the asset under consideration, making  $M_{t+1}$  “asset free”. This is typically the case for calculations with utility functions, which only involve one consumption good: there, one typically sets  $\tilde{C} = 1$  and interprets  $R_{t+1}$  as the real return in terms of that consumption good for investing one unit in it. More generally, one then typically sets  $\tilde{C}$  to be a unit vector, i.e. a vector containing only zeros except for one entry for a particular consumption good which is set to one, and likewise formulates  $R$  as random factor times this vector, so that  $R$  (or this random factor) can be interpreted as the return in terms of that particular consumption good.

### A.3 Asset pricing with Epstein-Zin preferences

The goal in Epstein and Zin (1989, 1991) was to derive asset pricing implications in terms of observable magnitudes, using their nonexpected utility specification. To do this, they assumed that there is a representative agent: in that case, “unobservables” such as utilities could be substituted out using data on market returns, etc.. Of course, there is no problem in principle to derive the Lucas asset pricing condition for an agent with Epstein-Zin preferences living in an arbitrary economy and enjoying some given stochastic stream of consumption: that asset pricing equation can be derived exactly as in the previous section. In the notation of the main text of this paper, one obtains after a brief calculation, that

$$M_{t+1} = \beta \left( \frac{(E_t [U_{t+1}^{1-\gamma}])^{\frac{1}{1-\gamma}}}{U_{t+1}} \right)^{\gamma-\sigma^{-1}} \left( \frac{C_t}{C_{t+1}} \right)^{\sigma^{-1}}. \quad (46)$$

Note that this collapses to the usual formula for  $M_{t+1}$  for the time separable case, if  $\gamma = \sigma^{-1}$ .

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